Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Find

(a) the coordinates of the foci of H,

(3)

(b) the equations of the directrices of H.

(2)

2.

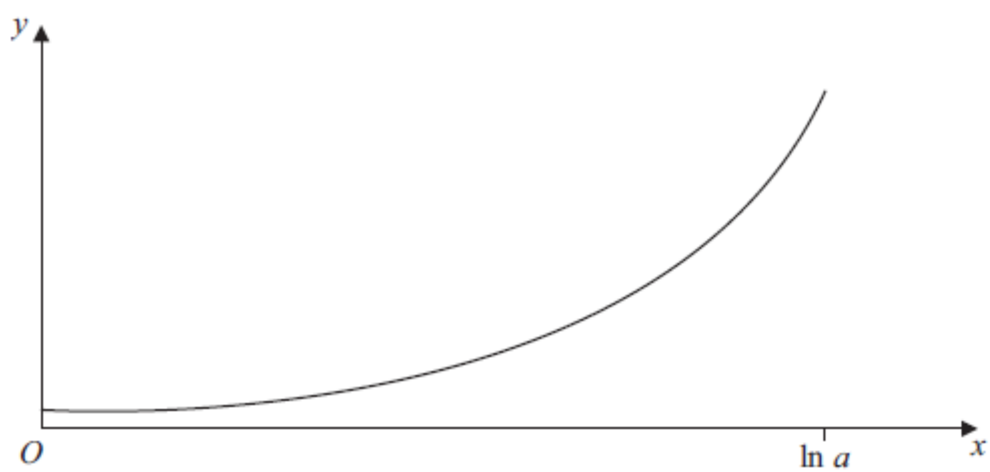

Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \leq x \leq \ln a,$$

where a is a constant and $a > 1$.

Using calculus, show that the length of curve C is

$$k\left(a^3 - \frac{1}{a^3}\right)$$

and state the value of the constant k.

(6)

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3. The position vectors of the points A, B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

(*a*) $\overrightarrow{AC} \times \overrightarrow{BC}$,

(4)

(*b*) the area of triangle ABC,

(2)

(*c*) an equation of the plane ABC in the form $\mathbf{r.\,n} = p$.

(2)

4.

$$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \; \mathrm{d}x, \quad n \geq 0.$$

(*a*) Prove that, for $n \geq 2$,

$$I_n = \frac{1}{4}n\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)\,I_{n-2}.$$

(5)

(*b*) Find the exact value of I_2.

(4)

(*c*) Show that $I_4 = \dfrac{1}{64}(\pi^3 - 24\pi + 48)$.

(2)

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5. (*a*) Differentiate x arsinh $2x$ with respect to x.

(3)

(*b*) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \; \mathrm{d}x.$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

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6. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

The line l_1 is a tangent to E at the point $P(a \cos \theta, b \sin \theta)$.

(*a*) Using calculus, show that an equation for l_1 is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1.$$

(4)

The circle C has equation

$$x^2 + y^2 = a^2.$$

The line l_2 is a tangent to C at the point $Q(a \cos \theta, a \sin \theta)$.

(*b*) Find an equation for the line l_2.

(2)

Given that l_1 and l_2 meet at the point R,

(*c*) find, in terms of a, b and θ, the coordinates of R.

(3)

(*d*) Find the locus of R, as θ varies.

(2)

7.

$$f(x) = 5 \cosh x - 4 \sinh x, \quad x \in \mathbb{R}.$$

(*a*) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$.

(2)

Hence

(*b*) solve $f(x) = 5$,

(4)

(*c*) show that $\displaystyle\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,dx = \frac{\pi}{18}$.

(5)

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8. The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}.$$

(*a*) Show that 4 is an eigenvalue of $\mathbf{M}$, and find the other two eigenvalues.

(5)

(*b*) For the eigenvalue 4, find a corresponding eigenvector.

(3)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\mathbf{M}$.

The equation of l_1 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(*c*) Find a vector equation for the line l_2.

(5)

TOTAL FOR PAPER: 75 MARKS

END

June 2012
6669 Further Pure Maths FP3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	Uses formula to obtain $e = \frac{5}{4}$	M1A1
	Uses ae formula	M1 (3)
(b)	Uses other formula $\frac{a}{e}$	M1
	Obtains both Foci are $(\pm 5, 0)$ and Directrices are $x = \pm\frac{16}{5}$ (needs both method marks)	A1 cso (2) (5 marks)

Notes

a1M1: Uses $b^2 = a^2(e^2 - 1)$ to get $e > 1$

a1A1: cao

a2M1: Uses ae

b1M1: Uses $\frac{a}{e}$

b1A1: cso for both foci and both directrices. Must have both of the 2 previous M marks may be implicit.

Question Number	Scheme	Marks
2.	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sinh 3x$	B1
	so $s = \int \sqrt{1+\sinh^2 3x}\,\mathrm{d}x$	M1
	$\therefore s = \int \cosh 3x\,\mathrm{d}x$	A1
	$= \left[\frac{1}{3}\sinh 3x\right]_0^{\ln a}$	M1
	$= \frac{1}{3}\sinh 3\ln a = \frac{1}{6}[\mathrm{e}^{3\ln a} - \mathrm{e}^{-3\ln a}]$	DM1
	$= \frac{1}{6}(a^3 - \frac{1}{a^3})$ (so $k = 1/6$)	A1 (6 marks)

Notes

1B1: cao

1M1: Use of arc length formula, need both $\sqrt{\ }$ and $\left(\frac{dy}{dx}\right)^2$.

1A1: $\int \cosh 3x\,\mathrm{d}x$ cao

2M1: Attempt to integrate, getting a hyperbolic function o.e.

3M1: depends on previous M mark. Correct use of lna and 0 as limits. Must see some exponentials.

2A1: cao

Question Number	Scheme	Marks
3. (a)	$\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $\overrightarrow{AC} \times \overrightarrow{BC} = 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}$	B1, B1 M1 A1 (4)
(b)	Area of triangle $ABC = \frac{1}{2}\lvert 10\mathbf{i} - 15\mathbf{j} + 30\mathbf{k}\rvert = \frac{1}{2}\sqrt{1225} = 17.5$	M1 A1 (2)
(c)	Equation of plane is $10x - 15y + 30z = -20$ or $2x - 3y + 6z = -4$ So **r.** (2**i** – 3**j** + 6**k)** = -4 or correct multiple	M1 A1 (2) (8 marks)

Notes

a1B1: $\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ cao, any form

a2B1: $\overrightarrow{BC} = -3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ cao, any form

a1M1: Attempt to find cross product, modulus of one term correct.

a1A1: cao, any form.

b1M1: modulus of their answer to (a) – condone missing ½ here. To finding area of triangle by correct method.

b1A1: cao.

c1M1: [Using their answer to (a) to] find **equation** of plane. Look for **a.n** or **b.n** or **c.n** for p.

c1A1: cao

Question Number	Scheme	Marks
4(a)	$I_n = \left[x^n(-\tfrac{1}{2}\cos 2x)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} -\tfrac{1}{2}nx^{n-1}\cos 2x\,dx$	M1 A1
	so $I_n = \left\langle \left[x^n(-\tfrac{1}{2}\cos 2x)\right]_0^{\frac{\pi}{4}} \right\rangle + \left[\tfrac{1}{4}nx^{n-1}\sin 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tfrac{1}{4}n(n-1)x^{n-2}\sin 2x\,dx$	M1 A1
	i.e. $I_n = \frac{1}{4}n\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)I_{n-2}$ *	A1cso
		(5)
(b)	$I_0 = \int_0^{\frac{\pi}{4}} \sin 2x\,dx = \left[-\tfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}} = \tfrac{1}{2}$	M1 A1
	$I_2 = \frac{1}{4}\times 2\times\left(\frac{\pi}{4}\right) - \frac{1}{4}\times 2\times I_0$, so $I_2 = \frac{\pi}{8} - \frac{1}{4}$	M1 A1 (4)
(c)	$I_4 = \left(\frac{\pi}{4}\right)^3 - \frac{1}{4}\times 4\times 3I_2 = \frac{\pi^3}{64} - 3\left(\frac{\pi}{8} - \frac{1}{4}\right) = \tfrac{1}{64}(\pi^3 - 24\pi + 48)$ *	M1 A1cso (2)

Notes

a1M1: Use of integration by parts, integrating $\sin 2x$, differentiating x^n.
a1A1: cao
a2M1: Second application of integration by parts, integrating $\cos 2x$, differentiating x^{n-1}.
a2A1: cao
a3A1: cso Including correct use of $\frac{\pi}{4}$ and 0 as limits.
b1M1: Integrating to find I_0 or setting up parts to find I_2.
b1A1: cao (Accept I_0 = ½ here for both marks)
b2M1: Finding I_2 in terms of π. If 'n''s left in M0
b2A1: cao
c1M1: Finding I_4 in terms of I_2 then in terms of π. If 'n''s left in M0
c1A1: cso

Question Number	Scheme	Marks
5. (a)	$\text{ar}\sinh 2x, + x\dfrac{2}{\sqrt{1+4x^2}}$	M1A1, A1 (3)
(b)	$\therefore \int_0^{\sqrt{2}} \text{arsinh}2x\text{d}x = \left[x\text{ar}\sinh 2x\right]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} \dfrac{2x}{\sqrt{1+4x^2}}\text{d}x$ $= \left[x\text{ar}\sinh 2x\right]_0^{\sqrt{2}} - \left[\dfrac{1}{2}(1+4x^2)^{\frac{1}{2}}\right]_0^{\sqrt{2}}$ $= \sqrt{2}\text{arsinh}2\sqrt{2} - [\frac{3}{2} - \frac{1}{2}]$ $= \sqrt{2}\ln(3+2\sqrt{2}) - 1$	1M1 1A1ft 2M1 2A1 3DM1 4M1 3A1 (7) (10 marks)

Notes

a1M1: Differentiating getting an arsinh term **and** a term of the form $\dfrac{px}{\sqrt{1 \pm qx^2}}$

a1A1: cao $\text{ar}\sinh 2x$

a2A1: cao + $\dfrac{2x}{\sqrt{1+4x^2}}$

b1M1: rearranging their answer to (a). **OR** setting up parts

b1A1: ft from their (a) **OR** setting up parts correctly

b2M1: Integrating getting an arsinh or arcosh term **and** a $\left(1 \pm ax^2\right)^{\frac{1}{2}}$ term o.e..

b2A1: cao

b3DM1: depends on previous M, correct use of $\sqrt{2}$ and 0 as limits.

b4M1: converting to log form.

b3A1: cao depends on all previous M marks.

Question Number	Scheme	Marks
6(a)	$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$ and so $\frac{dy}{dx}=-\frac{xb^2}{ya^2}=-\frac{b\cos\theta}{a\sin\theta}$ $\therefore\ y-b\sin\theta=-\frac{b\cos\theta}{a\sin\theta}(x-a\cos\theta)$ Uses $\cos^2\theta+\sin^2\theta=1$ to give $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ ✳	M1 A1 M1 A1cso (4)
(b)	Gradient of circle is $-\frac{\cos\theta}{\sin\theta}$ and equation of tangent is $y-a\sin\theta=-\frac{\cos\theta}{\sin\theta}(x-a\cos\theta)$ or sets $a=b$ in previous answer So $y\sin\theta+x\cos\theta=a$	M1 A1 (2)
(c)	Eliminate x or y to give $y\sin\theta(\frac{a}{b}-1)=0$ or $x\cos\theta(\frac{b}{a}-1)=b-a$ l_1 and l_2 meet at $(\frac{a}{\cos\theta}, 0)$	M1 A1, B1 (3)
(d)	The locus of R is part of the line $y=0$, such that $x\geq a$ and $x\leq -a$ Or clearly labelled sketch. Accept "real axis"	B1, B1 (2) (11 marks)

Notes

a1M1: Finding gradient in terms of θ. Must use calculus.
a1A1: cao
a2M1: Finding equation of tangent
a2A1: cso (answer given). Need to get $\cos^2\theta+\sin^2\theta$ on the same side.
b1M1: Finding gradient and equation of tangent, **or** setting $a=b$.
b1A1: cao need not be simplified.
c1M1: As scheme
c1A1: $x=\frac{a}{\cos\theta}$, need not be simplified.
c1B1: $y=0$, need not be simplified.
d1B1: Identifying locus as $y=0$ or real/'x' axis.
d2B1: Depends on previous B mark, identifies correct parts of $y=0$. Condone use of strict inequalities.

Question Number	Scheme	Marks
7(a)	$f(x) = 5\cosh x - 4\sinh x = 5\times\frac{1}{2}(e^{x}+e^{-x}) - 4\times\frac{1}{2}(e^{x}-e^{-x})$	M1
	$= \frac{1}{2}(e^{x}+9e^{-x})$ *	A1cso
		(2)
(b)	$\frac{1}{2}(e^{x}+9e^{-x}) = 5 \Rightarrow e^{2x} - 10e^{x} + 9 = 0$	M1 A1
	So $e^{x} = 9$ or 1 and $x = \ln 9$ or 0	M1 A1
		(4)
(c)	Integral may be written $\int \frac{2e^{x}}{e^{2x}+9}\,dx$	B1
	This is $\frac{2}{3}\arctan\left(\frac{e^{x}}{3}\right)$	M1 A1
	Uses limits to give $\left[\frac{2}{3}\arctan 1 - \frac{2}{3}\arctan(\frac{1}{\sqrt{3}})\right] = \left[\frac{2}{3}\times\frac{\pi}{4} - \frac{2}{3}\times\frac{\pi}{6}\right] = \frac{\pi}{18}$ *	DM1 A1cso
		(5)
		(11 marks)

Notes

a1M1: Replacing both cosh*x* and sinh*x* by terms in e^{x} and e^{-x} condone sign errors here.
a1A1: cso (answer given)
b1M1: Getting a three term quadratic in e^{x}
b1A1: cao
b2M1: solving to $x =$
b2A1: cao need ln9 (o.e) and 0 (not ln1)
c1B1: cao getting into suitable form, may substitute first.
c1M1: Integrating to give term in arctan
c1A1: cao
c2M1: Depends on previous M mark. Correct use of ln3 and ½ ln3 as limits.
c2A1: cso must see them subtracting two terms in π.

Question Number	Scheme	Marks
8. (a)	$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 0 \\ -1 & 0 & 4-\lambda \end{vmatrix} = 0 \therefore (2-\lambda)(2-\lambda)(4-\lambda)-(4-\lambda)=0$	M1
	$(4-\lambda)=0$ verifies $\lambda=4$ is an eigenvalue (can be seen anywhere)	M1
	$\therefore (4-\lambda)\{4-4\lambda+\lambda^2-1\}=0 \quad \therefore (4-\lambda)\{\lambda^2-4\lambda+3\}=0$	A1
	$\therefore (4-\lambda)(\lambda-1)(\lambda-3)=0$ and 3 and 1 are the other two eigenvalues	M1 A1 (5)
(b)	Set $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M1
	Solve $-2x+y=0$ and $x-2y=0$ and $-x=0$ to obtain $x=0, y=0, z=k$	M1
	Obtain eigenvector as **k** (or multiple)	A1 (3)
(c)	l_1 has equation which may be written $\begin{pmatrix} 3+\lambda \\ 2-\lambda \\ -2+2\lambda \end{pmatrix}$	B1
	So l_2 is given by **r** = $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 3+\lambda \\ 2-\lambda \\ -2+2\lambda \end{pmatrix}$	M1
	i.e. **r** = $\begin{pmatrix} 8+\lambda \\ 7-\lambda \\ -11+7\lambda \end{pmatrix}$	M1 A1
	So $(\mathbf{r}-\mathbf{c})\times\mathbf{d}=\mathbf{0}$ where **c** = 8**i** + 7**j** - 11**k** and **d** = **i** - **j** + 7**k**	A1ft (5) (13 marks)

Notes

a1M1: Condone missing = 0. (They might expand the determinant using any row or column)
a2M1: Shows λ = 4 is an eigenvalue. Some working needed need to see = 0 at some stage.
a1A1: Three term quadratic factor cao, may be implicit (this A depends on 1st M only)
a2M1: Attempt at factorisation (usual rules), solving to λ = .
a2A1: cao. If they state λ = 1 and 3 please give the marks.
b1M1: Using A***x*** = 4***x*** o.e.
b2M1: Getting a pair of correct equations.
b1A1: cao
c1B1: Using **a** and **b**.
c1M1: Using r = M x their matrix in **a** and **b**.
c2M1: Getting an expression for l_2 with at least one component correct.
c1A1: cao all three components correct
c2A1ft: ft their vector, must have **r =** or **(r-c)x** d = 0 need both equation and r.